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A MATHEMATICAL MODEL FOR THE COACH TRIP WITH SHUTTLE SERVICE PROBLEM

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A Mathematical Model for the Coach Trip with Shuttle Service Problem

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Abstract

In this work we consider the Coach Trip with Shuttle Service Problem (CTSSP), which is a routing problem where passengers have to be transported from bus stops to a central hub with a fleet of coaches and shuttles. The capacity of each vehicle must not be exceeded and for each group of passengers waiting at a bus stop there is a maximal travel time that must not be exceeded while travelling to the hub. Shuttles can use bus stops as transfer points to drop their passengers from which they have to be picked up by a coach. Coaches must end their trip at the hub while shuttles can stop at any bus stop. The goal is to minimize costs. The costs consist of travelling costs of the used vehicles plus fixed costs for the usage of the shuttles. We prove the computational complexity of the problem and present a novel mathematical model for the CTSSP. This model is implemented in CPLEX and the optimal solution of the "example" instance of the *VeRoLog Solver Challenge 2015* is shown.

Keywords: Coach Trip with Shuttle Service Problem, Transfers, Routing, CTSSP, VeRoLog Solver Challenge 2015, Computational Complexity, CPLEX

1 Introduction

The Coach Trip with Shuttle Service Problem (CTSSP) is a routing problem where, in preparation for a long distance journey, passengers have to be transported to a central hub. A huge variety of routing and transportation problems exists in the literature. For an overview we refer to [2], [6] and [7]. The CTSSP was presented by the *EURO Working Group on Vehicle Routing and Logistics Optimization (VeRoLog)* for the *VeRoLog Solver Challenge 2015* [3]. To our knowledge, it is a new optimization problem in the field of routing and transportation problems. From the existing problems, the School Bus Routing and Scheduling Problem with Transfers [1] is the problem with the most similarities to the CTSSP. It deals with the transportation of pupils from home to their school. It has a set of bus stops and a set of schools. Each pupil has walking and waiting times. The CTSSP differs with its unique hub instead of the set of schools as well as with its maximum travel times compared to the minimum and maximum waiting times before school begins. The *VeRoLog Solver Challenge 2015* inspired to research the problem. In particular the work of Geiger [5], who won the *VeRoLog Solver Challenge 2015* with his approach, is worthwhile to mention. He used a heuristic approach (Variable Neighborhood Search as well as Iterated Local Search) to solve the problem.

This paper contributes to this research domain by presenting a novel mathematical model for the CTSSP. We prove the computational complexity of the problem and implement the model in CPLEX.

This paper is structured as follows. First, a description of the problem is given in Section 2. Then, the computational complexity of the problem is proven (Section 3) and the mathematical model is presented (Section 4). In Section 5, the computational results of the CPLEX implementation are reported and conclusions are formulated in Section 6.

2 Problem Description

The Coach Trip with Shuttle Service Problem is a transportation problem, where passengers have to be transported to a central hub. The objective is minimizing the overall transportation costs. Each vehicle k in the fleet belongs either to the set of coaches C or the set of shuttles S. Every coach k starts at a specific location and has a fixed capacity. A coach trip can only visit a certain number of bus stops $S^k_{\max}$ (not counting the hub) and it must always end at the hub. Once a coach reaches the hub it stays there. Each group of passengers starts at a bus stop b_i and has a travel time limitation D_i that restricts the acceptable travel time of the group on its way to the hub. Additionally to coaches we can also rent shuttles to transport passengers either directly to the hub or to transfer points. A transfer point can be a bus stop with other passengers waiting or an empty bus stop. After a shuttle has dropped off its passengers at a transfer point the shuttle trip ends there and the passengers need to be picked up by exactly one coach. Passenger groups cannot be split.

The coach costs consist only of variable costs per distance travelled while for the shuttles there are additional fixed usage costs as they are booked extra. The vehicles categories "coach" and "shuttle" differ in speed and costs per travelled distance unit.

An instance of the Coach Trip with Shuttle Service Problem is given by a finite set of locations V that has the following disjoint subsets:

- The set B_+ of bus stops that need to be visited.
- The set B_0 of empty bus stops. These do not have passengers and can, but do not need to be visited.
- The set C of coaches.
- The set S of shuttles.

- The location H as the terminal hub.

For each bus stop $b_i \in B_+$, there is a number of waiting passengers $q_i \in \mathbb{Z}^+$. Every bus stop $b_i \in B_+ \cup B_0$ has a maximal travel time $D_i \in \mathbb{Z}^+$ to the hub (which is chosen sufficiently large for the empty ones). The coaches $k \in C$ start at their given location s_k with maximal capacity $cap_k \in \mathbb{Z}^+$, maximal number of stops $s^k_{\max} \in \mathbb{Z}^+$ and cost per travelled distance unit $varcost_k \in \mathbb{R}^+$. Similarly, for the shuttles $k' \in S$ the starting points are $s_{k'}$. Each shuttle has a capacity $cap_{k'} \in \mathbb{Z}^+$, cost per travelled distance unit $varcost_{k'} \in \mathbb{R}^+$ and usage cost $fixedcost_{k'} \in \mathbb{R}^+$. For every pair of locations $i, j \in V$ the travel distances from i to j for shuttles (and coaches) are given by $d^s_{i,j} \in \mathbb{Z}^+ (d^c_{i,j} \in \mathbb{Z}^+)$ and the travel times from i to j for shuttles (and coaches) are given by $t^s_{i,j} \in \mathbb{Z}^+ (t^c_{i,j} \in \mathbb{Z}^+)$.

The problem is to find a cost minimal routing of the coaches and shuttles such that the following conditions hold:

- All passengers are transported from locations in B_+ to the hub H.
- All coaches must travel to the hub H and cannot exceed stop limitations.
- Shuttles can drop off their passengers at any stop, but those must be picked up by a coach afterwards.
- The time limit of each passenger group must not be exceeded.
- Passenger groups cannot be split.
- Shuttles must not return to their start location.
- Shuttles can do at most one trip.
- The vehicle capacities must not be exceeded.

For each bus stop $b_i \in B_+$, there is a number of waiting passengers $q_i \in \mathbb{Z}^+$. Every bus stop $b_i \in B_+ \cup B_0$ has a maximal travel time $D_i \in \mathbb{Z}^+$ to the hub (which is chosen sufficiently large for the empty ones). The coaches $k \in C$ start at their given location s_k with maximal capacity $cap_k \in \mathbb{Z}^+$, maximal number of stops $s^k_{\max} \in \mathbb{Z}^+$ and cost per travelled distance unit $varcost_k \in \mathbb{R}^+$. Similarly, for the shuttles $k' \in S$ the starting points are $s_{k'}$. Each shuttle has a capacity $cap_{k'} \in \mathbb{Z}^+$, cost per travelled distance unit $varcost_{k'} \in \mathbb{R}^+$ and usage cost $fixedcost_{k'} \in \mathbb{R}^+$. For every pair of locations $i, j \in V$ the travel distances from i to j for shuttles (and coaches) are given by $d^s_{i,j} \in \mathbb{Z}^+ (d^c_{i,j} \in \mathbb{Z}^+)$ and the travel times from i to j for shuttles (and coaches) are given by $t^s_{i,j} \in \mathbb{Z}^+ (t^c_{i,j} \in \mathbb{Z}^+)$.

The objective is to find a feasible routing with minimal costs. The transportation costs consists of variable costs and of fixed costs. The variable costs depend on the distance travelled by the vehicles times the factor of their $varcost_k$. The fixed costs are usage costs per shuttle, if shuttles are used in the routing.

During the *VeRoLog Solver Challenge 2015* several instances were introduced. One of them is called “example” instance. To illustrate the problem, the “example” instance of the *VeRoLog Solver Challenge 2015* [3] is displayed in Figure 1.

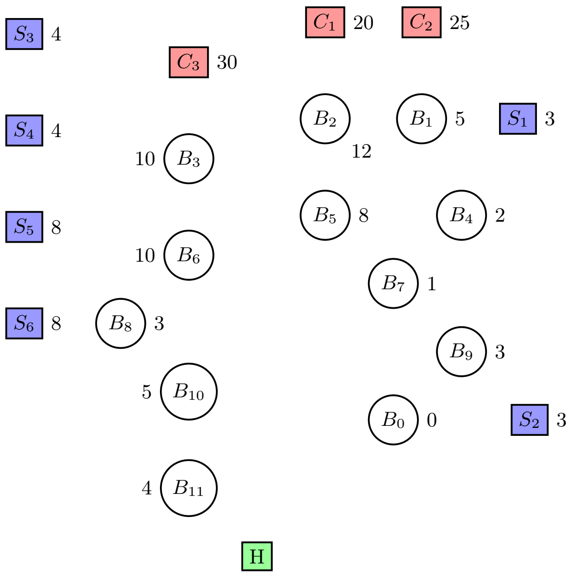

Figure 1: "Example" instance

The coaches are marked with C_1, C_2 and C_3, the shuttles got the labels $S_1, S_2, \ldots, S_6$. The distances in the figure do not correspond to the actual ones for illustration purpose (for example, coach 1 and coach 2 start at the same location). The labels next to the vehicle locations show their capacity while those next to the bus stops $B_0, B_1, \ldots, B_{11}$ display the quantity of waiting passengers at the location. The terminal hub is labelled with H.

Several optimal solutions for this problem can be found by using the mathematical model explained in Section 4 and presented in Section 5.

3 Computational Complexity

In this section a proof of the computational complexity of the CTSSP is given. Therefore, we introduce the decision variant CTSSP-D of the CTSSP. Here, for a given instance of CTSSP and a bound $b \in \mathbb{Z}^+$ it is to decide whether a solution with costs lower or equal than b exists or not.

Definition 3.1. (CTSSP-D) *Coach Trip with Shuttle Service Problem Decision variant*:
Instance: A CTSSP instance and bound $b \in \mathbb{Z}^+$.
Question: Does a feasible routing of the coaches and shuttles for the CTSSP instance exist such that the total costs cannot exceed b?

For the complexity proof we use the well known Directed Hamiltonian Path Problem with specific Start and End point (DHPSE) which is defined as follows.

Definition 3.2. (DHPSE) *Directed Hamiltonian Path with specific Start and End point*:
Instance: A graph $G = (V, A)$, start point $u \in V$, end point $v \in V$.
Question: Does G contain a directed Hamiltonian path from u to v, i.e., a directed path from u to v in G visiting all vertices in V exactly once?

Lemma 3.1. CTSSP-D is strongly NP-complete, even given unary notation of the parameters.
Proof: For a given solution of CTSSP-D it is trivial to check all constraints in polynomial time, so, CTSSP-D is in NP.

We will show that CTSSP-D is NP-complete with a reduction from DHPSE which is a NP-complete problem [4].

So, given an instance of DHPSE, i.e., a graph $G = (V, A)$ and two vertices $u, v \in V$ we construct an instance of CTSSP-D where all numerical parameters are bound by a polynomial of the input size $|V| = n$ in the following way: We set $V' := V \setminus \{u, v\}$ and for every $v_i \in V'$ we add a bus stop $b_i \in B_+$ with $q_i = 1$ and $D_i = 1$. The set C of coach stops will contain only one stop c_1 corresponding to the vertex u with capacity $cap_1 = n - 2$, maximal number of stops $s^1_{\max} = n - 2$ and cost per travelled distance unit $varcost_1 = 1$. The terminal hub H corresponds to the vertex v of the original graph. The set of empty bus stops B_0 and the set of shuttle start locations S are empty and, therefore, we set the travel times and distances of shuttles between locations to be 0. For the coaches the travel times between locations are also 0 and we define the distances between locations i and j with $v_i, v_j \in V$ corresponding to i, j, respectively, as follows:

$$d^c_{i,j} = \begin{cases} 1, & \text{if } (v_i, v_j) \in A \\ 2, & \text{else} \end{cases}$$

We set $b = n - 1$. It is easy to see that this transformation can be done in polynomial time.

Now, we prove that there exists a Hamiltonian $u - v$ path in the original graph G if and only if there is a feasible routing of the coach with costs no more than b.

Assume that there exists a Hamiltonian $u - v$ path $P = (u, v_1, \ldots, v_{n-2}, v)$ in G. We will construct a feasible coach trip that solves the CTSSP-D instance. Since P is a Hamiltonian path, it visits all vertices in V exactly once. We can route the coach according to the ordering of the path P and visit all bus stops. At each of the $n - 2$ stops in B_+ the coach picks up one passenger and satisfies the capacity constraint. In total, it stops at $n-2$ bus stops and the stop limitations are fulfilled. The time constraint for each bus stop is not violated since all travel times are set to 0 and each bus stop has a maximal travel time limit of 1. When the coach travels from location i to location j, it follows the ordering on the path P and, therefore, there exists an arc in G between the vertices corresponding to i and j. Hence, the total distance travelled by the coach is $n - 1$ and the total costs do not exceed b. The solution solves the CTSSP-D instance.

Conversely, assume that no Hamiltonian $u - v$ path exists for the given DHPSE instance. We show that every routing of the coach visiting all bus stops has costs greater than b. Each coach routing visiting all bus stops has to use at least one connection from a location $i \in B_+$ to a location $j \in B_+$ such that there is no arc between the corresponding vertices in G. If the arc would exist in G for all connections used on the routing, we could construct a Hamiltonian $u - v$ path in G which is a contradiction. So, the costs of each coach trip has to be at least $2 + n - 2 = n > b$ and, therefore, we cannot find a solution to the CTSSP-D instance.

This proves our claim. Since the parameters in the constructed instance are all bounded by n, CTSSP-D is NP-complete in the strong sense, even if we use unary notation to represent the parameters. □

4 Mathematical Model

In this section we formulate a mathematical model for the CTSSP. We add two dummy locations H' and TP' to the set of locations V. When a vehicle k uses less than the maximum number of its stops $S^k_{\max}$ it is allowed to circle in these dummy locations for the remaining stops. H' is a dummy hub and has a travel time and travel distance of 0 from and to the hub H. It is not reachable from any other location and cannot be left once entered. TP' is a dummy location needed for the transfer points and has a travel time and travel distance of 0 from all bus stops. Again, it is not reachable from any other location and cannot be left. We use the following variables:

$$x^{k,l}_{i,j} \in \{0,1\} \qquad \forall i,j \in V, \forall k \in C \cup S, l = 1,\ldots,S^k_{\max} \tag{1}$$

$$y^k \in \{0,1\} \qquad \forall k \in S \tag{2}$$

$$q^k \in \mathbb{Z}^+ \qquad \forall k \in C \cup S \tag{3}$$

$$t^k_l \in \mathbb{Z}^+ \qquad \forall k \in C \cup S, l = 1,\ldots,S^k_{\max} \tag{4}$$

$$z^{k,k'}_{i,l} \in \{0,1\} \qquad \forall i \in B_+ \cup B_0, \forall k \in C, \forall k' \in S, l = 1,\ldots,S^k_{\max} \tag{5}$$

$$\tilde{q}^{k,k'} \in \mathbb{Z}^+ \qquad \forall k \in C, \forall k' \in S \tag{6}$$

$$\tilde{t}^{k',k} \in \mathbb{Z}^+ \qquad \forall k' \in S, \forall k \in C \tag{7}$$

The variable $x^{k,l}_{i,j}$ in (1) is set to 1 if and only if vehicle k enters location j as the l-th stop on its trip after visiting location i before. The variable y^k in (2) indicates for each shuttle k if it is used in the routing or not. Variable q^k in (3) is set to the number of passengers transported by vehicle k. Each variable t^k_l in (4) is set to the time needed from the l-th stop on the trip of vehicle k to the last stop. The variable $z^{k,k'}_{i,l}$ in (5) is equal to 1 if and only if there is a transfer point at location i for shuttle k' and coach k and the coach enters location i at the l-th position of its trip. Here, the position l of the transfer point location on the coach trip is needed for the $\tilde{t}$ variables in (7). Variable $\tilde{q}^{k,k'}$ in (6) is set to 0 if shuttle k' and coach k do not share a transfer point and are set to the number of passengers transported by shuttle k' if they do. Finally, variable $\tilde{t}^{k',k}$ in (7) is set to 0 if shuttle k' and coach k do not share a transfer point and are set to the travel time from the transfer point to the hub of the coach if they do.

Hence, we use the following objective function to minimize the sum of variable costs and the sum of fixed costs for renting shuttles:

$$\min \sum_{k \in C \cup S} varcost_k \cdot \left(\sum_{l=1}^{S^k_{\max}} \sum_{i,j \in V} x^{k,l}_{i,j} \cdot d^k_{i,j} \right) + \sum_{k \in S} y^k \cdot fixedcost_k \tag{8}$$

The set $B = B_+ \cup B_0 \cup \{H, H', TP'\}$ contains all bus stops, the hub H and the dummy locations H' and TP'. We begin by introducing the constraints that ensure that trips start at the right locations and that they are connected. For every vehicle $k \in C \cup S$, we denote with s_k the start location of vehicle k.

$$\sum_{j \in B} x^{k,1}_{s_k,j} = 1 \qquad \forall k \in C \tag{9}$$

$$\sum_{j \in V} x^{k,1}_{i,j} = 0 \qquad \forall k \in S, \forall i \in V \setminus \{s_k\} \tag{10}$$

$$\sum_{j \in V \setminus B} x^{k,l}_{i,j} = 0 \qquad \forall k \in C \cup S, l = 1, \ldots, S^k_{\max}, \forall i \in V \tag{11}$$

$$\sum_{i,j \in V} x^{k,1}_{i,j} \leq y^k \qquad \forall k \in S \tag{12}$$

$$\sum_{i,j \in V} x^{k,l}_{i,j} \leq 1 \qquad \forall k \in C \cup S, l = 1, \ldots, S^k_{\max} \tag{13}$$

$$\sum_{j \in V} x^{k,l}_{j,i} = \sum_{j \in V} x^{k,l+1}_{i,j} \qquad \forall k \in C \cup S, l = 1, \ldots, S^k_{\max} - 1, \forall i \in V \tag{14}$$

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^k_{\max}} \sum_{\substack{j \in V \\ j \neq H', TP'}} x^{k,l}_{i,j} \leq 1 \qquad \forall i \in V \tag{15}$$

Constraints (9) ensure that all coaches start from their start location and visit a bus stop location at their first stop. By (10) it is prohibited that a shuttle can use any other location than its start location for a start. With the constraints in (11) no vehicle can visit the start locations of other vehicles (these are the locations in $V \setminus B$). Constraints (12) force the variable y^k to 1 if shuttle k is used. With constraints (13) it is ensured that each vehicle k can only go at most from one location to one other location at each position l of its trip. Constraints (14) ensure that the trip of each vehicle is connected. If the vehicle enters a location i at the l-th position of its trip, it has to leave this location i when going to the $(l+1)$-th position of its trip. In (15), for every location i at most one vehicle can leave this location to another location j except for the two dummy locations. Note that it is possible for multiple vehicles to leave location

i if they visit the transfer point dummy location TP' or the dummy hub H' as next location. This is necessary since in case of a transfer point between a coach and a shuttle both have to visit the same location but only the coach carries on its trip to the hub H whereas the shuttle goes to the transfer point dummy location TP' afterwards. Also, in case of the hub H there may be more than one vehicle leaving it to the dummy hub location.

Next, we present some constraints necessary for the dummy locations.

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^k_{\max}} \sum_{j \in V \setminus \{H'\}} x^{k,l}_{H,j} = 0 \tag{16}$$

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^k_{\max}} \sum_{j \in V \setminus \{H,H'\}} x^{k,l}_{j,H'} = 0 \tag{17}$$

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^k_{\max}} \sum_{j \in V \setminus \{H'\}} x^{k,l}_{H',j} = 0 \tag{18}$$

$$\sum_{k \in S} \sum_{l=1}^{S^k_{\max}} \sum_{j \in V \setminus \{TP'\}} x^{k,l}_{TP',j} = 0 \tag{19}$$

By constraint (16) no vehicle can travel from the hub location H to any other location except the dummy hub location H'. It is prohibited by constraint (17) that the dummy hub location H' can be reached by any other location than the actual hub location H or the dummy hub location itself. Constraint (18) ensures that from the dummy hub location H' vehicles can only travel to the dummy hub location, i.e., they are forced to cycle there once they reached it. Similarly, constraint (19) forces shuttles that enter the transfer point dummy location to cycle there and prohibits them to travel to another location.

The following constraints ensure that each trip ends in a feasible way and all bus stops with customers are visited by a vehicle.

$$\sum_{l=1}^{S^k_{\max}} \sum_{i \in V} x^{k,l}_{i,H} = 1 \qquad \forall k \in C \tag{20}$$

$$\sum_{l=1}^{S^k_{\max}} \sum_{i \in V} x^{k,l}_{i,H} + \sum_{k' \in C} \sum_{l'=1}^{S^{k'}_{\max}} \sum_{j \in B_+ \cup B_0} z^{k',k}_{j,l'} = y^k \qquad \forall k \in S \tag{21}$$

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^k_{\max}} \sum_{j \in V \setminus \{TP'\}} x^{k,l}_{i,j} = 1 \qquad \forall i \in B_+ \tag{22}$$

Constraints (20) force every coach k to visit the hub location H on its trip. Note that it does not need to be the last stop of its trip since the coach can travel to the dummy hub location H' afterwards. If a shuttle k is used, the right hand side of (21) is equal to 1 and forces the left hand side of the equation to be 1 as well: Either the shuttle travels to the hub at some point of its trip (left hand sum) or the shuttle shares a transfer point with some coach k' (right hand sum). By constraints (22) it is ensured that for each bus stop i with customers waiting there is exactly one vehicle leaving the bus stop to another location differing from the transfer point dummy location.

Next, the constraints necessary for the transfer point variable z are introduced.

$$\sum_{j\in V} x_{j,i}^{k,l} \geq z_{i,l}^{k,k'} \qquad \forall k\in C, \forall k'\in S, \forall i\in B_+\cup B_0, l=1,\ldots,S_{\max}^k \tag{23}$$

$$\sum_{l'=1}^{S_{\max}^{k'}} \sum_{j\in V} x_{j,i}^{k',l'} \geq z_{i,l}^{k,k'} \qquad \forall k\in C, \forall k'\in S, \forall i\in B_+\cup B_0, l=1,\ldots,S_{\max}^k \tag{24}$$

$$\sum_{j\in V} x_{j,i}^{k,l} + \sum_{l'=1}^{S_{\max}^{k'}} \sum_{j'\in V} x_{j',i}^{k',l'} - 1 \leq z_{i,l}^{k,k'} \qquad \forall k\in C, \forall k'\in S, \forall i\in B_+\cup B_0, l=1,\ldots,S_{\max}^k \tag{25}$$

Constraints (23) force $z_{i,l}^{k,k'}$ to be equal to 0 if coach k does not enter location i as its l-th stop of the trip. Similarly, if shuttle k' does not enter location i at any position of its trip, constraints (24) force $z_{i,l}^{k,k'}$ to be equal to 0. In constraints (25), if both shuttle k' and coach k visit location i, then it is used as a transfer point and the corresponding $z_{i,l}^{k,k'}$ variable is forced to be equal to 1. The constraints are always fulfilled if only one of the vehicles enters the location.

To ensure the maximum travel time for each customer, the following constraints are introduced (where T is an integer number greater than the maximum travel time for any vehicle).

$$\sum_{l'=l+1}^{S_{\max}^k} \sum_{i,j\in B} x_{i,j}^{k,l} \cdot t_{i,j}^c = t_l^k \qquad \forall k \in C, l = 1,\ldots,S_{\max}^k \tag{26}$$

$$\sum_{l'=l+1}^{S_{\max}^k} \sum_{i,j\in B} x_{i,j}^{k,l} \cdot t_{i,j}^s = t_l^k \qquad \forall k \in S, l = 1,\ldots,S_{\max}^k \tag{27}$$

$$t_l^k \leq \sum_{i\in V} \sum_{j\in B_+\cup B_0} x_{i,j}^{k,l} \cdot D_j \qquad \forall k \in C\cup S, l = 1,\ldots,S_{\max}^k \tag{28}$$

$$\tilde{t}^{k',k} \leq T \cdot \sum_{l=1}^{S_{\max}^k} \sum_{i\in B_+\cup B_0} z_{i,l}^{k,k'} \qquad \forall k \in C, \forall k' \in S \tag{29}$$

$$\tilde{t}^{k',k} - t_l^k \leq (1 - \sum_{i\in B_+\cup B_0} z_{i,l}^{k,k'}) \cdot T \qquad \forall k \in C, \forall k' \in S, l = 1,\ldots,S_{\max}^k \tag{30}$$

$$\tilde{t}^{k',k} - t^k_l \geq (1 - \sum_{i \in B_+ \cup B_0} z^{k,k'}_{i,l}) \cdot (-T) \quad \forall k \in C, \forall k' \in S, l = 1, \ldots, S^k_{\max} \quad (31)$$

$$t^{k'}_l + \sum_{k \in C} \tilde{t}^{k',k} \leq \sum_{i \in V} \sum_{j \in B_+ \cup B_0} x^{k',l}_{i,j} \cdot D_j \qquad \forall k' \in S, l = 1, \ldots, S^{k'}_{\max} \quad (32)$$

Constraints (26) set the value of t^k_l to the travel time needed from the l-th stop on the trip of coach k to the last stop. Similarly, in (27), the travel times for shuttles from their l-th stop to the last stop are set. Constraints (28) ensure that for each bus stop j with customers waiting on the trip of vehicle k the maximum travel time D_j is not exceeded. Constraints (29) – (31) are used to set the $\tilde{t}$ variables to the correct value. In (29), the $\tilde{t}$ variable is forced to be 0 if there is no transfer point for coach k and shuttle k'. If there is a transfer point at location i at the l-th stop of coach k and shuttle k', the Inequalities (30) and (31) enforce $\tilde{t}^{k',k} = t^k_l$, i.e., the $\tilde{t}$ variable is set to the travel time from the transfer point to the last stop of the coach. Finally, in (32) it is ensured that, if a transfer point is used, the combined travel time of shuttle and coach does not exceed the maximum travel time for each bus stop j with customers waiting on the trip of a shuttle. If no transfer points are used on the shuttle trip, all $\tilde{t}$ variables are 0 and the constraint is the same as in (28).

The following terms ensure that vehicle capacities are not violated.

$$\sum_{l=1}^{S^k_{\max}} \sum_{i \in B_+} \sum_{j \in V \setminus \{TP'\}} x^{k,l}_{i,j} \cdot q_i = q^k \qquad \forall k \in C \cup S \quad (33)$$

$$q^k \leq cap_k \qquad \forall k \in C \cup S \quad (34)$$

$$\tilde{q}^{k,k'} \leq cap_{k'} \cdot \sum_{l=1}^{S^k_{\max}} \sum_{i \in B_+ \cup B_0} z^{k,k'}_{i,l} \qquad \forall k \in C, \forall k' \in S \quad (35)$$

$$\tilde{q}^{k,k'} - q^{k'} \leq (1 - \sum_{l=1}^{S^k_{\max}} \sum_{i \in B_+ \cup B_0} z^{k,k'}_{i,l}) \cdot cap_{k'} \qquad \forall k \in C, \forall k' \in S \quad (36)$$

$$\tilde{q}^{k,k'} - q^{k'} \geq (1 - \sum_{l=1}^{S^k_{\max}} \sum_{i \in B_+ \cup B_0} z^{k,k'}_{i,l}) \cdot (-cap_{k'}) \qquad \forall k \in C, \forall k' \in S \quad (37)$$

$$\tilde{q}^{k,k'} \leq q^{k'} \qquad \forall k \in C, \forall k' \in S \quad (38)$$

$$q^k + \sum_{k' \in S} \tilde{q}^{k,k'} \leq cap_k \qquad \forall k \in C \quad (39)$$

In Constraints (33), the variable q^k is set to the amount of passengers waiting at locations that are visited by vehicle k. Note, that only the connections that do not end in the dummy transfer point location TP' are counted. This is sufficient since for shuttles that use a bus stop as a transfer point the passengers there do not need to fit in the shuttle because they are picked up by the coach. By (34) it is ensured that the capacity of the vehicles is not exceeded. Constraints

(35) – (38) set the value of $\tilde{q}$ correctly. If coach k and shuttle k' do not share a transfer point, by (35) the variable $\tilde{q}^{k,k'}$ is forced to be 0. However, if they use a transfer point (i.e., the corresponding z variable is 1 for some location i and some position l), we enforce $\tilde{q}^{k,k'} = q^{k'}$ by (36) and (37), i.e., the $\tilde{q}$ variable is set to the amount of passengers on the shuttle. Constraints (38) ensure that the value of the variable $\tilde{q}^{k,k'}$ can not exceed the value of the variable $q^{k'}$ for a shuttle k'. With constraints (39) for all coaches also the total number of passengers must not violate the capacity of the coach (if no transfer point is used the constraint is the same as in (34)).

In total, there are $|V|^2 \cdot (|C| + |S|) \cdot S_{max}^{max} + |S| + |V|^2 \cdot |C| \cdot |S| \cdot S_{max}^{max}$ binary decision variables and $|C| + |S| + (|C| + |S|) \cdot S_{max}^{max} + 2 \cdot |C| \cdot |S|$ integer valued decision variables (where S_{max}^{max} is the maximum number of stops over all vehicles). The number of constraints is bounded by $(|C| + |S|)^2 \cdot |V| \cdot S_{max}^{max}$.

5 Computational Experiments

The implementation of the model was made with CPLEX 12.6.3. Three datasets were presented during the competition phase of the *VeRoLog Solver Challenge 2015*. The first instance was the "example" instance with 3 coaches, 6 shuttles and 13 bus stops (including the hub). Another two instances called "public_preselection_1" and "public_preselection_2" had 6 coaches, 333 shuttles and 144 bus stops each. They differ in terms of the properties of the bus stops. During the competition phase three additional datasets were released. A detailed overview of the properties of all datasets is given by Geiger [5].

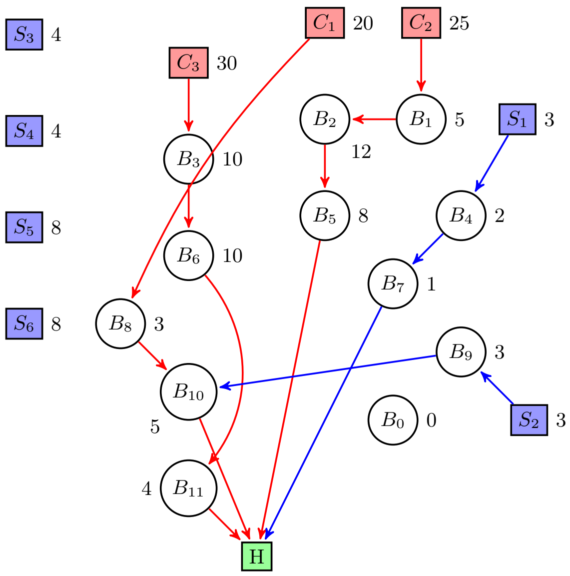

Figure 2: First optimal solution for the "example" instance

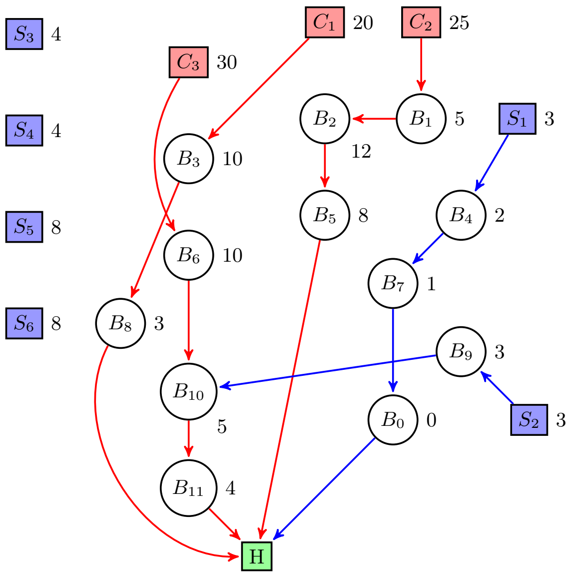

Figure 3: Second optimal solution for the "example" instance

The "example" instance was solved within a few seconds. For the other two benchmark instances, CPLEX was not able to find a solution at all due to memory limitations. The optimal solution of the "example" instance has an objective value 417. According to CPLEX, at least two optimal solutions with the value of 417 exist.

They are displayed in Figure 2 and Figure 3 and are quite similar. They only differ in the routing of coach C_1 and coach C_3, as well as in the routing of shuttle S_1. The route of C_1 in the first solution is $B_8 \to B_{10} \to H$, while the route of C_1 in solution 2 is $B_3 \to B_8 \to H$. While coach C_3 visits $B_3 \to B_6 \to B_{11} \to H$ in solution 1, it approaches $B_6 \to B_{10} \to B_{11} \to H$ in solution 2. The shuttle once takes the direct way from B_7 to the hub (solution 1) and once the empty bus stop B_0 in between before going to the hub. Both trips of shuttle S_1 have equal length. This example shows that the triangle inequalities do not necessarily hold for the data sets. Obviously, at least two more optimal solutions exist – shuttle 1 could be routed over bus stop B_0 in the first optimal solution and vice versa in the second one.

6 Conclusions

We presented a mathematical model for this novel vehicle routing problem and analyzed its computational complexity. For some preliminary computational experiments this mathematical formulation was implemented in CPLEX. For rather small instances CPLEX was able to find optimal solutions quickly. However, for a growing number of bus stops and vehicles the solver was not able to find optimal solutions due to memory limitations.

An alternative way to find good and feasible solutions for this problem is the application of heuristics (see the approach of Geiger [5]). Another possible approach is the use of a matheuristic. Based on a feasible start solution suitable subproblems can be identified and solved to optimality using the presented mathematical model. This should be investigated in future research.

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