How to quantify the maturity of production processes

Negar Arabizadeh[1], **Julius Pfrommer**[2], **and Jürgen Beyerer**[1,2]
[1]Karlsruhe Institute of Technology, Vision and Fusion Laboratory (IES), Karlsruhe, Germany
[2]Fraunhofer IOSB, Karlsruhe, Germany

Abstract

Quantifying the properties of interest is a basic need in all fields of science and engineering. We propose formal definitions to quantify the maturity of production processes. The definitions are inspired by the concepts of controllability and observability from control theory. But instead of a binary classification of a system as controllable/uncontrollable and observable/unobservable, we consider a probability distribution for the remaining error. From the proposed maturity quantification, the remaining potential for optimizations can be evaluated. This is shown for the example of an electric arc furnace (EAF), for which high-fidelity simulation models are available.

Keywords Immature production processes · Maturity metrics for production processes · Complex system modeling

1 Introduction

A manufacturing process is immature if it cannot operate with the required productivity and product quality. New manufacturing processes involve a development phase before they are capable of production with the required quality, productivity, and cost-effectiveness. Abstractly speaking, a huge and complex optimization problem must be solved during the development phase in terms of the process structure, parameter values, control strategies, and process instrumentation (sensors and actuators).

Having metrics for measuring the maturity of a production process is essential, as it provides an overview of the process's development status in comparison with the desired goals of a mature process. Existing research on evaluating the maturity of production processes is mainly based on qualitative models that provide little or no quantitative meaning. On the other hand, quantitative models in the literature are primarily focused on measuring the Technology Readiness Level (TRL) rather than maturity [MMSM13]. Furthermore, there are no quantitative models that relate the internal features of the production processes (process variables and process dynamics) to the desired outcome of the production process, which is the quality of the final product. This gap motivated us to introduce new definitions to quantify the maturity of production processes, considering both the quality of the final product and the internal features of the process, and to investigate a relationship between them (such as a monotonic relation).

A qualitative metric in [Met11] presents a maturity assessment model to measure the maturity of social and technical systems. Also, due to the lack of knowledge on how to design theoretically sound and widely accepted maturity assessment models, the paper discusses the typical phases of maturity model development and application, taking a design science research perspective. In [Pfl95], a qualitative metric plan was proposed, which outlines five levels of process maturity and the corresponding types of metrics required for each level. In existing research, different fields have developed quantitative measures for their areas of interest. In [BG16], a quantitative definition of risk for safety and security was developed. In [Bey99], a metric, Bayesian decision theory, was proposed to evaluate the relative worth of additional knowledge within the measurement context.

The scientific contribution of this paper is as follows:

- Definition of the maturity of a production process.
- Introducing quantitative definitions for measuring the maturity of production processes with the eventual goal of being able to relate the internal features of the dynamic system to the quality of the eventual product of the production process.
- Implementing our new definitions for quantifying maturity in an example of an electric arc furnace (EAF) model.

2 Related works

2.1 Maturity models in literature

In [PCCW93], the immature organization, especially in the context of software processes, is described as a reactionary organization that focuses on solving immediate crises, is not cost-efficient, is unscheduled, and, especially under tight deadlines, is unable to meet desired product quality and functionality. It has no objective way to judge the quality of the product or solve product or process problems, making the quality of production difficult to predict. As a result, activities aimed at improving quality are curtailed or eliminated. In literature, maturity is defined as "the state of being complete, perfect, or ready" [SS^{+}04]. For production processes, we can say that it shows evolutionary progress toward the target of the production process or the demonstration of a specific ability [Met11]. In [PCCW93], a "matured organization" is defined as an organization that follows the scheduled deadlines to prepare products and achieve the desired quality goals. It has management, less waste, the potential to improve, and is economically acceptable. Models for measuring the maturity of production processes on an ordinal scale are available in existing research. For example, [WK02] presents a maturity model, but it does not offer a sound mathematical definition for measuring maturity on a ratio scale. In [Met11] the concept of maturity assessment models for social systems was contextualized. In the literature, the terms "maturity" and "readiness" are sometimes used interchangeably, but there are some differences between them [SM96].

2.2 Generalizing controllability and observability

Classical definitions of controllability and observability in control theory

The original definition of controllability and observability from the control theory literature is as follows [Kal60]:

Controllability: The system state $\mathbf{s}$ is controllable if there exists an input signal $\mathbf{x}(t)$ over a finite time interval $0 \leq t \leq t_1$ such that a target state $\check{\mathbf{s}}$ is reached at t_1. When a state of the system is controllable, it is possible to control the system state to a desired point.

Observability: The system state $\mathbf{s}$ is observable if the state $\mathbf{s}(0)$ at time $t = 0$ can be fully determined by observations $\mathbf{y}(t)$ during a finite time interval $t_0 \leq t \leq 0$.

There are different methods in the literature, such as calculating the rank of the controllability matrix and the controllability Gramian matrix [Kal63], which indicate whether the system is controllable or not without providing any answer about the degree of controllability. Methods like [HN89, HS80, Tar92, VLL84] measure the degree of controllability of a system without any physical meaning. Therefore, measures based on minimum input energy are mostly used to give the degree of controllability a physical meaning.

Extending the classical definitions of controllability and observability

[MW72] proposed an infinite set of possible scalars to measure the degree of controllability and observability for linear dynamical systems. The physical interpretation of these measures is the minimum input energy required to regulate a system from its initial condition over a finite time interval. If the system requires less energy, it can be considered more controllable. [LP16] proposed a method to measure the degree of controllability of a system with unstable modes, based on the physical meaning of measuring the minimum energy required to change the state from an arbitrary initial condition to an arbitrary final condition within a positive time interval. Since the measure is related to the initial and final conditions, the appropriate initial and final conditions corresponding to the control objective should be used.

In an application, these measures were used to optimize certain structural parameters in order to improve the proposed quality measures. [MKG02, MKG04] proposed Gramian-based methods to determine the optimal locations of sensors and actuators. [SH05, SH06] extended these methods to nonlinear systems. In [KPPS09], the degree of controllability is defined as the minimum input energy required to achieve the final state zero in the presence of persistent external disturbance. It is composed of Gramian controllability and sensitivity matrices. In [ST24], VCS (Volumetric Controllability Score) and AECS (Average Energy Controllability Score) are defined. These are two mathematical definitions based on the Gramian controllability matrix and are unique solutions to the convex optimization problem, producing a degree for measuring the controllability of a system. In other words, the VCS of each state node indicates its importance in enlarging the controllability ellipsoid, and the AECS of each state node indicates its importance in steering overall states to a point on the unit sphere. This metric measures the energy input needed to change the state from one condition to the sphere. The characteristic function of the VCS is based on the log determinant of the controllability Gramian matrix, and the AECS is the trace of the inverse of this matrix. By the rule of duality, they could also provide insights into observability. [XYC$^+$18] introduced a measure for the degree of controllability of a linear system with random initial conditions and disturbance, by solving the problem of expected minimum energy transfer in fixed time and applying the method in turbines. In [SLM17], they developed a new data-driven method to measure the degree of controllability of a system using data.

The classical definitions of controllability and observability provide a definitive binary answer as to whether a system is controllable or observable. However, the classical definitions and their extensions in the existing research do not account for how much we are able to control or observe the system with a specific level of precision, considering the uncertainty. For example, in the case of controllability, the classical definitions do not account for how much we are able to control the system states to achieve their desired states with specific precision (similarly for observability). In the real world, uncertainty (such as output/input noise, process noise, etc.) prevents us from achieving the exact desired state with zero error. Instead, we can only reach the desired state within a certain level of precision. By considering uncertainty, we can evaluate the probability of successfully controlling the system states to the desired states with a specific degree of precision. A similar approach applies to observability, in terms of how much we are able to observe the states of the system with a specific level of precision.

By considering uncertainty and incorporating probability, we can better describe how effectively the system can be controlled or observed. Instead of simply stating whether the system is controllable or observable, we can assess how effectively we can bring the system states to their desired states or observe the system states with a specific level of precision. In this context, not only do we answer the questions of controllability and observability, but we also gain insights into the precision of these properties.

Consequently, an extension of these concepts is necessary, as we wish to capture the stochastic nature of production processes and the quality of their output. In ML, the concept of PAC learning (Probably Approximately Correct) [Val84] describes the number of data samples minimally required to achieve a model error smaller than some η with a probability larger than $1 - \delta$. We apply similar notions for generalizing the definitions of controllability and observability, considering the uncertainty that exists in the system and the precision we want to achieve to be able to quantify the maturity of the production processes.

3 Quantifying process maturity

As discussed in the previous section, there are different factors to consider when quantifying process maturity. For example, these include the ability to generate profit, the quality of the final product, the speed of the process, and the consumption of resources. Some aspects, such as the speed and quality of the process, depend on the product and can be used to quantify maturity, while others are related to the market conditions. The following is our new definition of a "mature production process":

Definition 1. *A production process is mature if it cannot be further improved economically.*

The economic argument is not really related to the physical production process itself, and we rather focus on measurable quantities for the physical production process and not for the market conditions, which are beyond the scope of this research.

As long as there is potential to improve the production process to meet the required threshold (considering its specific applications and economic aspects), it cannot be considered mature. This is not only due to technical considerations but also economic factors. Therefore, we integrate both aspects into our definition of maturity. In this context, we can determine the threshold at which the process is deemed sufficient, taking into account both its specific applications and economic feasibility.

For people in industry, the internal dynamics of the system is often not easily understood, comprehensible, or considered important or interesting. Their main concern is the economic aspect of the production process. In general, a mature system is one that prioritizes the economic efficiency of production.

The difference from previous definitions in the literature is that they focus on various factors, such as the efficiency of the production process, adherence to the schedule, and product quality. Since all of these factors ultimately impact the economic aspect of the production process, we can conclude that a system is mature when it is economically optimized. We introduce new definitions to quantify the maturity of production processes that relate the quality of the final product to its internal features.

3.1 Notation and definitions

The transformation of initially immature production processes into a higher maturity is achieved through a sequential approach of process development and subsequent expansion of production [FSL03]. Figure 1 shows the evolution of process development in which we present process instances after making software or hardware changes. If we make hardware changes, we move from the process instance P_i^j to the process instance P_{i+1}^j, and if we make parameter or software changes, we move from the process instance P_i^j to the process instance P_i^{j+1}. Changes can sometimes be done in parallel. After making these changes, there might be a possibility of achieving our desired mature process instance P^* or aborting, as shown by $\times$ in the figure. Figure 2 illustrates an idealized version of the closed-loop automated process instance P_i^j. It is made up of two main parts. One part shows the process variables and process dynamics, the input signal $\mathbf{x}$, noise $\mathbf{n}$ that might occur in the system, the state variables $\mathbf{s}$ of the dynamic system, the parameters of the system $\boldsymbol{\theta}$, the controller π and the output signal $\mathbf{y}$ which is a function of $\mathbf{x}, \mathbf{s}, \boldsymbol{\theta}$, and $\mathbf{n}$. The signals $\mathbf{x}$ and $\mathbf{y}$ are defined to be observable and $\mathbf{s}$ subsumes all unobservable dynamic quantities. If an added sensor or actuator instrumentation allows the components of $\mathbf{s}$ to be determined passively or actively, these quantities are assigned to be additional components of $\mathbf{x}$ and $\mathbf{y}$ in the next process instance P_{i+1}^j. The other part illustrates the input process material ε such as reactants and components with properties $\boldsymbol{\phi}$ and its flow to the product p with its properties $\mathbf{q}(p)$.

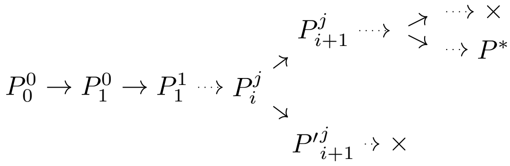

Figure 1: Evolution of the maturation of a production process. The symbol × indicates abortion, and P^* denotes the desired final mature process instance (subscripts denote the version with respect to structure, and superscripts enumerate soft improvements). If promising, it is conceivable to split the process maturation evolution into competing, parallel paths that pursue alternative process instances, here indicated as P'.

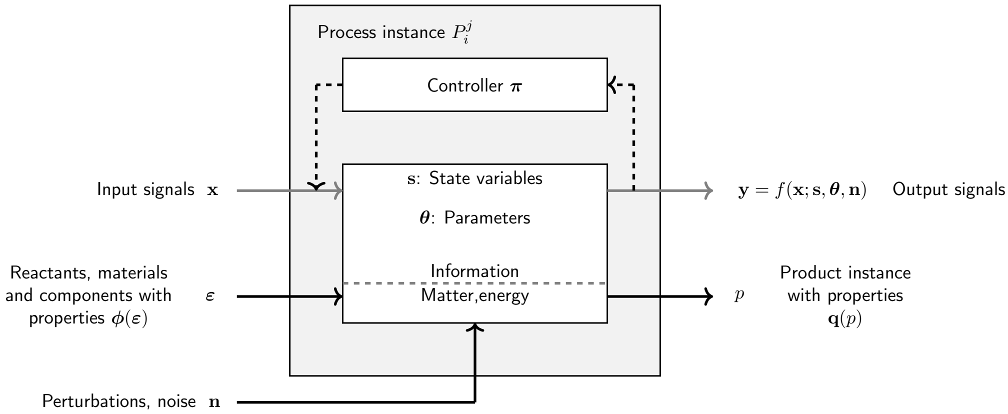

Figure 2: An abstract view of an idealized version of the closed-loop automated process instance. This may represent the complete production process or a sub-process at any hierarchical process level. The variables $\mathbf{x}$, $\mathbf{y}$, $\mathbf{n}$, and $\mathbf{s}$ are time-dependent signals. In fact, all involved variables should be marked with the indices ^j_i, since they refer to the process instance P_i^j, but this is suppressed whenever the context is clear.

3.2 New definitions to quantify process maturity

Elucidability

From here we consider the maturity of a process instance. For simplicity, we refer to it as P instead of P_i^j or do not even mention it, when the meaning is clear from context. For the process instance P_i^j, Elucidability which is a generalization of observability, is approached by defining the minimum observation time τ that is necessary to achieve a certain estimation precision for the state $\mathbf{s}$ and the parameters $\boldsymbol{\theta}$. We then find the probability of having the estimation error less than a constant precision $\eta_{\boldsymbol{\theta}}$ and $\eta_{\mathbf{s}}$. The internal states of the process can be estimated from the time series $\mathcal{X}_T$, $\mathcal{Y}_T$ during the finite observation duration T. Thus, $\mathcal{X}_T := \mathbf{x}(0), ..., \mathbf{x}(T)$ and $\mathcal{Y}_T := \mathbf{y}(0), ..., \mathbf{y}(T)$. The formula for Elucidability is:

$$E(T, \eta_s, \eta_{\boldsymbol{\theta}}) := \Pr(\|\mathbf{s}(T) - \hat{\mathbf{s}}(\mathcal{X}_T, \mathcal{Y}_T)\| < \eta_s \wedge \|\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(\mathcal{X}_T, \mathcal{Y}_T)\| < \eta_{\boldsymbol{\theta}} | \mathbf{s}(0)) \tag{1}$$

Here $\hat{\mathbf{s}}(\mathcal{X}_T, \mathcal{Y}_T)$ is the estimation of the state $\mathbf{s}(T)$ at time T, with time series $\mathcal{X}_T$ and $\mathcal{Y}_T$ over duration T. The same applies to $\hat{\boldsymbol{\theta}}(\mathcal{X}_T, \mathcal{Y}_T)$, which is an estimation of $\boldsymbol{\theta}$. If the time duration T is not fixed for a given process instance, an appropriate *Dual Elucidability*, inspired from PAC learning [Val84], is defined as the minimum time to achieve a required error with a probability higher than $1 - \delta$.

$$\tau_E(\eta_{\mathbf{s}}, \eta_{\boldsymbol{\theta}}, \delta) := \min\{T > 0 | E(T, \eta_{\mathbf{s}}, \eta_\theta) > 1 - \delta\}. \tag{2}$$

Assuming that the Elucidability monotonically increases with the time T, by construction, E can be recovered from a characterization of τ_E and vice versa. Note that τ_E tends to infinity if a required estimation error cannot be achieved for any duration of the observations.

Forcability

Forcability F, which is the generalization of controllability, is defined to quantify the ability to steer the values of $\mathbf{y}$ and $\mathbf{s}$ via the control input $\mathbf{x}$ towards the target values $\check{\mathbf{y}}$ and $\check{\mathbf{s}}$. The most effective way to steer the process is to establish a closed-loop control system that reacts instantaneously to the dynamic responses of the process. To quantify the Forcability of a process instance with a given controller π, the probability of being able to force the process after time T into the neighborhood of target values $\check{\mathbf{y}}$ and $\check{\mathbf{s}}$ is defined as follows:

$$F_\pi(T, \eta_\mathbf{s}, \eta_\mathbf{y}) := \Pr(\|\mathbf{y}(T) - \check{\mathbf{y}}\| < \eta_y \wedge \|\mathbf{s}(T) - \check{\mathbf{s}}\| < \eta_\mathbf{s} \,|\, \mathbf{s}(0), \mathbf{x}(t) = \pi(\mathcal{Y}_t), \check{\mathbf{y}}, \check{\mathbf{s}}) \quad (3)$$

In closed loop control, $\mathcal{X}_T$ is determined by the control policy π. In the case where we do not have $\mathbf{y}(T)$ and $\mathbf{s}(T)$ and need to estimate them, we can replace $\mathbf{y}(T)$ and $\mathbf{s}(T)$ by the estimates $\hat{\mathbf{y}}(T)$ and $\hat{\mathbf{s}}(T)$. Thus, the modified formula for an estimation of Forcability is:

$$\hat{F}_\pi(T, \eta_\mathbf{s}, \eta_\mathbf{y}) := \Pr(\|\hat{\mathbf{y}}(T) - \check{\mathbf{y}}\| < \eta_\mathbf{y} \wedge \|\hat{\mathbf{s}}(T) - \check{\mathbf{s}}\| < \eta_\mathbf{s} \,|\, \mathbf{s}(0), \mathbf{x}(t) = \pi(\mathcal{Y}_t), \check{\mathbf{y}}, \check{\mathbf{s}}) \quad (4)$$

To achieve a final statement about Forcability, the optimal control strategy $\mathbf{x}(t) = \pi^*$ has to be applied.

$$F(T, \eta_\mathbf{s}, \eta_\mathbf{y}) := F_{\pi^*}(T, \eta_\mathbf{s}, \eta_\mathbf{y}) \quad (5)$$

With that, analogous to Elucidability, Forcability is defined as the probability of achieving target values $\check{\mathbf{y}}$ and $\check{\mathbf{s}}$ with deviation smaller than $\eta_\mathbf{y}$ and $\eta_\mathbf{s}$ applying the optimal control strategy π^*. Also in analogy to Elucidability, the *Dual* of Forcability is the minimum time required to achieve the desired remaining error under the regime of the optimal controller π^* with a sufficiently high probability $1 - \delta$.

$$\tau_F(\eta_\mathbf{s}, \eta_\mathbf{y}, \delta) = \min\{T > 0 | F(T, \eta_\mathbf{s}, \eta_\mathbf{y}) > 1 - \delta\}. \quad (6)$$

Supervisability

Since the product instances p delivered by the process and their quality are the ultimate objectives of production, it seems reasonable to base a maturity measure on the quality of the product. Supervisability S is defined to measure the maturity of the process instance P_i^j. Based on the probability of $\mathbf{q}(p)$ falling in the desired set Q, given that we have set our controller π and the input signal $\mathbf{x}$, the state $\mathbf{s}$ and the parameter $\boldsymbol{\theta}$ remain in their admitted sets. Supervisability is defined as:

$$S(P_i^j) := \Pr(\mathbf{q} \in Q | \mathbf{x} \in X_\mathrm{adm}, \mathbf{s} \in S_\mathrm{adm}, \boldsymbol{\theta} \in \Theta_\mathrm{adm}, T < T_\mathrm{max}). \quad (7)$$

where X_adm, S_adm, and Θ_adm are the sets of admitted values for $\mathbf{x}$, $\mathbf{s}$, and $\boldsymbol{\theta}$, respectively, and the throughput time T necessary for producing a product instance does not exceed T_max. S measures the maturity of a process only from the technical point of view. To determine whether the resulting optimized process P^* is economic, the cost of the process must also be considered. Applying the economic aspect is beyond the scope of this publication, but will be considered in future work.

4 Numerical example: electric arc furnace

We used an electric arc furnace (EAF) system modeled in [BLA03] as a numerical example to showcase our definitions. An EAF system is the steelmaking process that uses the heat of an electric arc to melt the iron. According to studies in the literature, in which three-phase EAF systems with three electrodes were modeled, the general model of the system is in this form $\mathbf{i}(\mathbf{s}_\mathrm{pos}, \mathbf{u}) : \mathbb{R}^3 \to \mathbb{R}^3$, where the inputs are the positions of each electrode in meters (m) and the outputs are the currents of each electrode in ampere (A). The relationship between current and position of each electrode is explained in more detail in the Appendix A.

4.1 Controller design

One of the quality features considered for the EAF system is the temperature. It is assumed that heat transfer to the liquid metal is fast enough so that the temperature of the liquid metal is uniform throughout [BCP99]. A similar situation applies to liquid slag.

There are differences in temperature between the melted part of the furnace, the scrap and the solid part of the slag. The melted part heats both the scrap and the solid part of the slag. The heating rate is proportional to the difference between the temperature of the solid part and the temperature of the liquid part.

We consider that the liquid part is heated by the heat it receives from the arc of the electrodes, so the formula for the rate of change of temperature is:

$$\frac{ds_{\text{temp}}}{dt} = \frac{Q_{arc}}{mc_p} \tag{8}$$

where s_{temp} is the temperature in Kelvin, and we want to achieve 1773 K as the target temperature. Q_{arc} is the power received from the electrodes, m is the mass of the liquid steel, and c_p is the heat capacity of the liquid steel. According to [LDŠ12], $c_p = 0.84$ KJ/KgK, and we assume $m = 80$ t and the temperature starts at 298 K which is the temperature of the room at this temperature $c'_p = 0.45$ KJ/KgK. We produce Q_{arc} by calculating the formulas below.

The formula for the power of all three electrodes is $Q_{arc} = \mathbf{u}^\top \mathbf{i}$, $\quad E_{arc} = \int_0^T Q_{arc} dt + E_{start}$, where $\mathbf{u}$ and $\mathbf{i}$ are vectors of RMS values of voltage and current of each electrode, and E_{start} is calculated with $E_{start} = mc'_p \Delta s_{\text{temp}} = 10.728$ GJ, where $\Delta s_{\text{temp}} = 298$ K. We know the eventual energy that we want to achieve from the eventual temperature that is our desired temperature. Considering the initial energy (E_{start}), which is the energy we have at room temperature (298 K) we can calculate the difference energy ($\int_0^T Q_{arc} dt$) that we have to inject into the EAF system.

We consider that in the EAF model explained in the previous section, we have output noise. Therefore, we added sensor noise $\mathcal{N}(0\ \text{A}, 500\ \text{A}^2)$ to the measurement of the output current. In addition, we have initial condition noise $\mathcal{N}(0\ \text{m}, 0.1\ \text{m}^2)$ for the positions of the electrodes. Therefore, we solved the stochastic differential equation (SDE) for the relationship between the output current and the position of the electrodes:

$$d\mathbf{s}_{\text{pos}} \propto (\check{\mathbf{s}}_{\text{pos}} - \mathbf{s}_{\text{pos}}(t))dt + 0.05 dW_t \tag{9}$$

where W_t denotes a standard Brownian motion [GDHK11]. We designed a controller to achieve the desired product quality, i.e. a steel temperature of 1773 K. The input values that can be set are the positions of the electrodes. Furthermore, the electric current can be switched off once at an appropriate time (we do not allow bang-bang control – switching on/off repeatedly – during operations).

First, we compute the power required from the arc furnace to achieve the desired temperature after the duration T. This results directly from Equation (8). From this we compute $\mathbf{i}_{\text{des}}$, so that the target power is achieved and the maximum current that flows through any of the electrodes is minimal.

Second, recall that the mathematical model of the EAF predicts the current $\mathbf{i}$ resulting from the admissible positions of the electrodes $\mathbf{s}_{\text{pos}} \in S_{\text{adm}}$. The final position of the electrodes is selected as the solution of an optimization problem. This optimization problem is solved statically at the start of operations.

$$\min_{\mathbf{s}_{\text{pos}} \in S_{\text{adm}}} \|\mathbf{i}_{\text{des}} - \mathbf{i}(\mathbf{s}_{\text{pos}})\| \tag{10}$$

Here $\mathbf{i}_{\text{des}}$ is the vector of desired currents that we want to achieve for each electrode, and $\mathbf{i}(\mathbf{s}_{\text{pos}})$ is the vector of current of each electrode which is function of its position. The set of admissible values for the position of each electrode, S_{adm} is $[0\ \text{m}, 1.5\ \text{m}]^3$, as considered in [BLA03].

Third, a dynamic state-feedback controller at runtime tries to achieve the target position of the electrodes (accounting for any noise) and performs a controlled shut-off of the electric current when the desired temperature is achieved (under sensor noise for the temperature sensors).

4.2 Maturity quantification for the electric arc furnace system

Forcability plot

For calculating the Forcability considering the state of the system, which is position $\mathbf{s}_{\text{pos}}$ we have to calculate this probability:

$$F_\pi(T, \eta_{s^1_{\text{pos}}}) := \Pr(\|s^1_{\text{pos}}(T) - \breve{s}^1_{\text{pos}}\| < \eta_{s^1_{\text{pos}}} \,|\, s^1_{\text{pos}}(0), x(t) = \pi(Y_t), \breve{s}^1_{\text{pos}}) \tag{11}$$

It takes 10 s (ramp-up time) for the EAF to reach the desired current of 10 kA. Thus, the time duration is $T = 10$ s. As the calculation for position of all of the electrodes are the same we just illustrate the Forcability for the position of the first electrode s^1_{pos}, so $\eta_{s^1_{\text{pos}}} \in [0\text{ m}, 0.5\text{ m}]$ is the precision(error) that we want to achieve. $s^1_{\text{pos}}(0)$ has the form that we represent in Section 4.1 in the (SDE) so $s^1_{\text{pos}}(0) = 0\text{ m} + \mathcal{N}(0\text{ m}, 0.1\text{ m}^2)$, controller π is the controller for achieving the desired current which was explained in Section 4.1(9), and the equation (14) in the Appendix B, and considering the solution of the optimization problem (10), the desired position is $\breve{s}^1_{\text{pos}} = 0.79$ m.

To empirically calculate the probability, we run the simulation 1000 times. In each iteration, for each specific $\eta_{s^1_{\text{pos}}}$ we counted errors $\|s^1_{\text{pos}}(T) - \breve{s}^1_{\text{pos}}\|$ less than it at different durations $T \in \{0\text{ s}, 1\text{ s}, .., 10\text{ s}\}$. Finally, we divided the count value corresponding to each $\eta_{s^1_{\text{pos}}}$ for each specific duration T by 1000 to find the probability of having an error less than it. Figure 3 illustrates this Forcability plot.

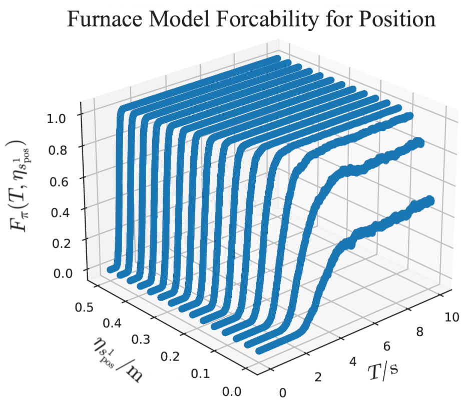

Figure 3: The Forcability plot for the position of the first electrode of the EAF model s^1_{pos} illustrates the probability of steering s^1_{pos} to its desired value with specific precision, as presented in (11).

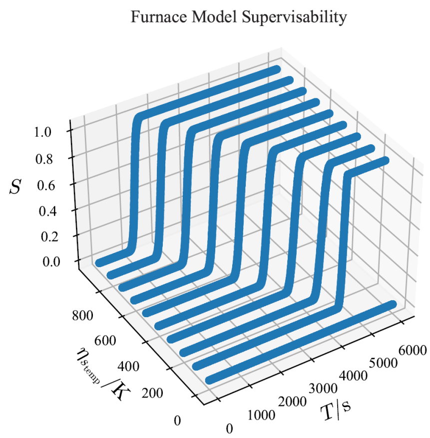

Figure 4: Supervisability plot for achieving the desired temperature as illustrated in equation (12).

As expected, essentially the probability increases with increasing error $\eta_{s^1_{\text{pos}}}$ and time T monotonically. The noises in the lines shows the uncertainty caused by sensor noise and the output noise of the system.

Supervisability plot

For calculating the Supervisability considering the temperature s_{temp} as the feature of quality we have to calculate the probability:

$$S := \Pr(\|s_{\text{temp}}(T) - \breve{s}_{\text{temp}}\| < \eta_{s_{\text{temp}}} \,|\, \mathbf{s}_{\text{pos}} \in [0\text{ m}, 1.5\text{ m}]^3, T < 10\text{ h}) \tag{12}$$

where the time duration $T = 6000$ s should be in the acceptable range which is $T < 10$ h, $\breve{s}_{\text{temp}} = 1773$ K is the desired temperature that we want to achieve at the end of time duration T, $\eta_{s_{\text{temp}}} \in [0\text{ K}, 1000\text{ K}]$ is the precision (error) that we want to achieve. $\mathbf{s}_{\text{pos}} \in [0\text{ m}, 1.5\text{ m}]^3$ represents the vector of electrode positions and should be in its admissible set $S_{\text{adm}} = [0\text{ m}, 1.5\text{ m}]^3$.

Estimating S is accomplished analogously to calculating the Forcability in the previous part, and we run the simulation 1000 times. In each iteration, for each specific $\eta_{s_{\mathrm{temp}}}$ we counted errors $\|s_{\mathrm{temp}}(T) - \breve{s}_{\mathrm{temp}}\|$ less than it at different durations $T \in \{0\ \mathrm{s}, 1\ \mathrm{s}, .., 6000\ \mathrm{s}\}$. Finally, we divided the count value corresponding to each $\eta_{s_{\mathrm{temp}}}$ for each specific duration T by 1000 to find the probability of having an error less than it. Figure 4 illustrates this Supervisability plot. As expected, the probability increases with increasing time and error $\eta_{s_{\mathrm{temp}}}$ monotonically.

5 Summary and outlook

In this paper, we introduced new definitions to quantify the maturity of a production process. We introduced Elucidability and Forcability which are generalization of the classical definitions in control theory: observability and controllability. E quantifies the ability to observe the states and parameters of the system, while F quantifies the ability to steer the outputs and states of the system, which are related to the internal dynamics of the system. S represents the probability of achieving the desired quality for the final product. Using the example of an EAF system, we illustrate E, F, and S in a practical example. The presented probability plots can be compared to assess how changes in the process can increase its maturity. In the future, we plan to investigate the impact of hardware and software changes on E, F, and S and to explore whether there is a monotonic relationship between these quantities.

Additionally, Figure 2 illustrates an idealized version of the closed-loop automated process instance, excluding networks of processes or manual processes with human intervention. In the future, we can explore distributed production processes, where each part may be located in different geographic locations, as well as processes that involve human intervention, decision-making, and other human factors. Moreover, taking into account the increased complexity of the system, especially when attempting to calculate S in a system where quality metrics $\mathbf{q}(p)$ are subjective or context-dependent.

Acknowledgments

This project is funded by DFG FOR 5339.

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A EAF detail

Figure 5 is the physical model of the EAF system [BLA03]. The relation between the position

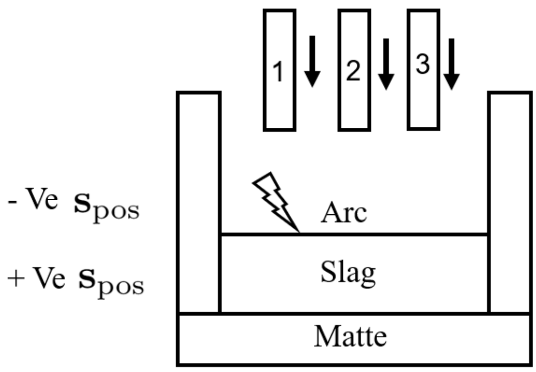

Figure 5: The illustration of three phase EAF system [BLA03]

and the current of each electrode is:

$$\mathbf{i} = \mathbf{G}(\mathbf{s}_{\text{pos}}, \mathbf{u})\mathbf{s}_{\text{pos}} + \mathbf{b}(\mathbf{s}_{\text{pos}}, \mathbf{u})$$

$\mathbf{i}$ is a 3×1 vector shows the current of each electrode, $\mathbf{s}_{\text{pos}}$ is a 3×1 vector shows the position of each electrode, $\mathbf{G}$ is the 3×3 conductance matrix, and $\mathbf{b}$ is the 3×1 constant vector and both are function of voltage and position. g_k denotes the slag to matte conductance and its relation with the position of each electrode is:

$$g_k = c_k s^k_{\text{pos}} + g_s$$

where $k = 1, 2, 3$ denotes the three different electrodes, and c_k is the conductance coefficient (in siemens (S)/m)), s^k_{pos} is the position of each electrode (in m) and g_s is the conductance of the slag when the electrodes are positioned at the surface of the slag (in S). In modeling the system from the physical model to the electric circuit, the matte is considered the virtual ground, so the voltage of this node in the electric circuit $u_m = 0$ V. In addition, the inter-electrode conductance is considered the same for all three electrodes and is denoted by g. The dynamic equation of the system is:

Table 1: Table of variables in the EAF system, we reproduce the values from the original paper

Variables	Acceptable Range	Chosen Value
u_1, u_2, u_3	$100 - 1000$ V	500 V
c_1, c_2, c_3	$1 - 100$ S/m	20 S/m
g_s	$5 - 25$ S	10 S
g	$[0, 0.15]$ S	0.1 S

$$\mathbf{G} = \frac{1}{g_{tot}} \begin{bmatrix} \begin{matrix} 2u_1c_1(g_s+g) - u_2c_1(g_s+g) \\ -u_3c_1(g_s+g) + c_1c_2s^2_{\text{pos}}(u_1-u_2) \\ +c_1c_3s^3_{\text{pos}}(u_1-u_3) \end{matrix} & \begin{matrix} u_1c_2(g_s+2g) - u_2c_2(g_s+g) \\ -u_3c_2g \end{matrix} & \begin{matrix} u_1c_3(g_s+2g) - u_2c_3g \\ -u_3c_3(g_s+g) \end{matrix} \\ \begin{matrix} -u_1c_1(g_s+g) + u_2c_1(g_s+2g) \\ -u_3c_1g \end{matrix} & \begin{matrix} -u_1c_2(g_s+g) + 2u_2c_2(g_s+g) \\ -u_3c_3(g_s+g) + c_2c_3s^3_{\text{pos}}(u_2-u_3) \\ +c_1c_2s^1_{\text{pos}}(u_2-u_1) \end{matrix} & \begin{matrix} -u_1c_3g - u_2c_3(g_s+2g) \\ -u_3c_3(g_s+g) \end{matrix} \\ \begin{matrix} -u_1c_1(g_s+g) - u_2c_1g \\ -u_3c_1(g_s+2g) \end{matrix} & \begin{matrix} -u_1c_2g - u_2c_2(g_s+g) \\ +u_3c_2(g_s+2g) \end{matrix} & \begin{matrix} -u_1c_3(g_s+g) - u_2c_3(g_s+g) \\ 2u_3c_3(g_s+g) + c_1c_3s^1_{\text{pos}}(u_3-u_1) \\ +c_2c_3s^2_{\text{pos}}(u_3-u_2) \end{matrix} \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \mathbf{G} \begin{bmatrix} s^1_{\text{pos}} \\ s^2_{\text{pos}} \\ s^3_{\text{pos}} \end{bmatrix} + \frac{g_s^2 + 3gg_s}{g_{tot}} \begin{bmatrix} 2u_1-u_2-u_3 \\ -u_1+2u_2-u_3 \\ -u_1-u_2+2u_3 \end{bmatrix} \tag{13}$$

where the u_1, u_2, and u_3 are the voltage of each electrode and $g_{tot} = c_1s^1_{\text{pos}} + c_2s^2_{\text{pos}} + c_3s^3_{\text{pos}} + 3g_s$. Table 1 shows the typical values for this EAF system as a three-phase electric circuit. The voltages are phasors with 500 V amplitude and 120° phase difference, so for each electrode we have $u_1 = u_2 = u_3 = 500$ V, $c_1 = c_2 = c_3 = 20$ S/m, $g_s = 10$ S and $g = 0.1$ S.

B Dynamic equation of the system

The dynamic equation for the system is related to the relation of the position and velocity of the electrodes:

$$\frac{d\mathbf{s}_{\text{pos}}}{dt} \propto \mathbf{s}_{\text{pos}}(t)$$

where $\mathbf{s}_{\text{pos}}$ (vector of the positions of the electrodes in meter) is the state of the system.

For controlling the system we consider that we have a state-feedback controller, so the dynamic equation of the closed loop system is in the form (ODE):

$$d\mathbf{s}_{\text{pos}} \propto (\check{\mathbf{s}}_{\text{pos}} - \mathbf{s}_{\text{pos}}(t))dt$$

where $\check{\mathbf{s}}_{\text{pos}}$ is the desired set point. Considering the noise that accrues in the system instead of (ODE) we will have a (SDE) of the form:

$$d\mathbf{s}_{\text{pos}} \propto (\check{\mathbf{s}}_{\text{pos}} - \mathbf{s}_{\text{pos}}(t))dt + 0.05dW_t \tag{14}$$

where W_t denotes a standard Brownian motion [GDHK11].