# Full Range Dynamic Equivalent Circuit Model for PEM Fuel Cell Stacks

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This work presents the development and parameterization of a dynamic equivalent circuit model of proton exchange membrane (PEM) fuel cell stacks. The model represents the static and dynamic behavior of the fuel cell stack under pulsed ohmic loading up to switching frequencies of 10 kHz. The model is developed using established mathematical descriptions of fuel cell processes. The model is then parameterized and validated using measurement data obtained from an 11 cell, 110 W PEM fuel cell stack.

Hydrogen fuel cell systems are a promising option as power sources for the transport sector. Here, and especially in the aerospace sector, they provide the possibility for emission-free propulsion, while still enabling energy and power densities required for relevant payloads and mission ranges. Thus, a major challenge of aerospace system design is the system mass and volume. [1] shows that PEM fuel cell stacks may be loaded with a high-frequency inverter current ripple without significant effects on lifetime. It allows optimizing the DC grid between fuel cell and propulsion inverter, which is a subject of ongoing research [2]. This requires dynamic fuel cell models representing the entire dynamic range up to common power electronic switching frequencies. [3] proposes a dynamic model for up to 100 Hz. This work aims to develop and present a model which is validated for pulse frequencies up to 10 kHz. Commonly used equivalent impedances, such as constant phase elements, were avoided to provide seamless transfer between frequency and time domain.

Measurements were conducted on a PEM fuel cell stack with an active area of 31.5 cm² and 11 cells. The stack is operated at 50 °C with an anode fuel flow of 2.07 NLPM and a cathode fuel flow of 7.93 NLPM. The stack was controlled using a Greenlight Innovation G100 test station. Polarization curves were obtained using an electronic load. Current pulses were applied using a custom circuit board directly attached to the stack, to minimize parasitic inductance. Low inductance thick film resistors were used to preload the fuel cell stack and as pulsed load, switched by a power-MOSFET. The gate-driver was controlled with a variable frequency square-wave generator. Measurement data was obtained using an oscilloscope, a current probe, and a differential voltage probe. Data was obtained at square-wave frequencies of 1 Hz, 10 Hz, 100 Hz, 1 kHz, and 10 kHz separately and is shown in Figure 1. Higher measured pulse frequencies up to 100 kHz were dominated by parasitic inductive effects. The measured polarization curve is shown in Figure 2.

Modeling of the PEM fuel cell stack is done in MATLAB/Simulink/Simscape using standard circuit components such as voltage sources, resistors, and capacitors. The thermodynamic cell potential $E_{\text{therm}}$ is modeled using an ideal voltage source. Different overvoltage mechanisms are modeled using current-controlled variable resistors. Time-dependent effects, as the double-layer capacity and diffusion effects, are modeled using capacitors. The activation overvoltage $\eta_{\text{act}}$ is described using the commonly applied Butler-Volmer equation or the Tafel equation with the coefficients $A$, $B$, and the current $i$. A current dependent activation resistance $R_{\text{act}}$ follows as (1). Then, the conduction and transport losses with ohmic behavior are modeled using a fixed resistance $R_{\text{ohmic}}$. The diffusion processes, limiting the currents in the high current regions, are modeled using a diffusion overvoltage $\eta_{\text{diff}}$ following the simplified Nernst-Equation according to [4]. A current-dependent

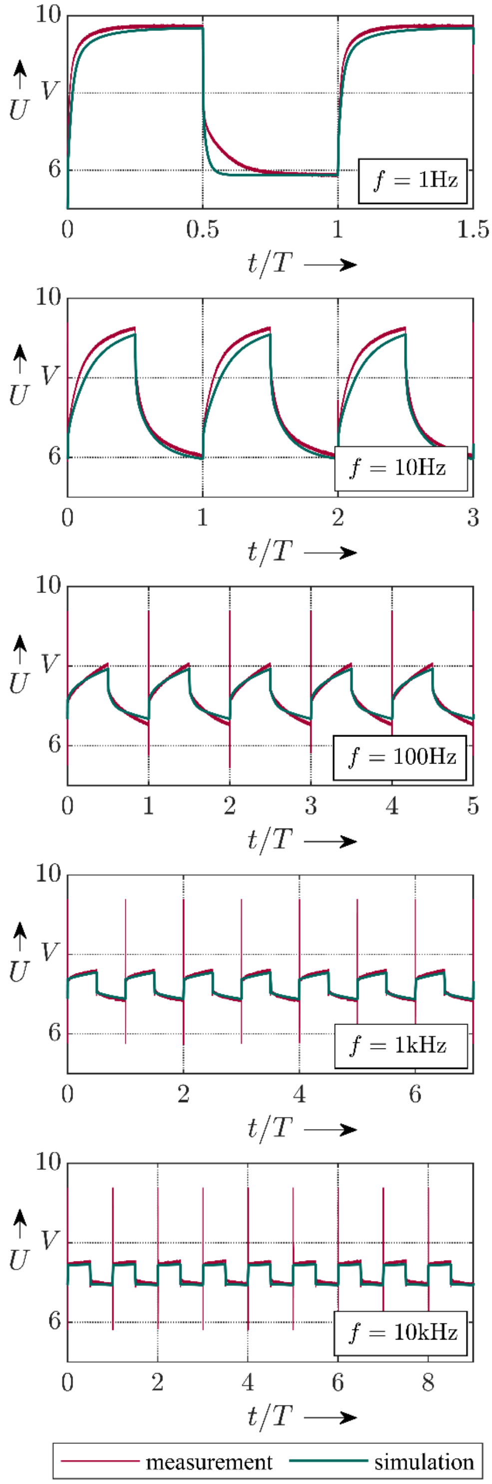


**Figure 1**. Measured and simulated stack voltages

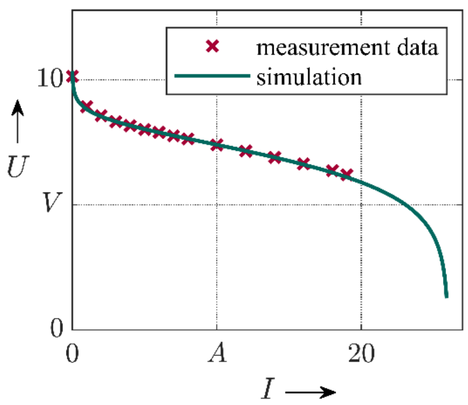


**Figure 2**. Measurement data and simulated polarization curve

diffusion resistance $R_{\text{diff}}$ is derived as (2) using a coefficient $D$ and the current limit $i_{\text{limit}}$. Additionally, a leakage current $i_{\text{leak}}$ is introduced using an ideal current source to represent parasitic electrochemical reactions and effects that lower the cell potential, especially in the low current region. The effective stack current is then given by $i = i_{\text{leak}} + i_{\text{load}}$. The terminal voltage $U$ of the PEM fuel cell stack is given as a function of stack current $i$ as (3).

$$R_{\text{act}} = \frac{\eta_{\text{act}}}{i} = \frac{A + B\ln(i)}{i} \tag{1}$$

$$R_{\text{diff}} = \frac{\eta_{\text{diff}}}{i} = \frac{D\ln\left(\frac{i_{\text{limit}}}{i_{\text{limit}} - i}\right)}{i} \tag{2}$$

To represent the full dynamic range of the fuel cell stack, the model was extended to a Ladder-Structure as proposed by [5] and is shown in Figure 3. Over the commonly used Voigt's-Structure, the Ladder-Structure has the advantage of charged capacitances for no-load scenarios. Commonly used equivalent circuit components, like a Warburg impedance or other constant phase elements, were avoided to allow for a seamless transfer between frequency and time domain. The equivalent capacitances $C_{dl}$ represent the double-layer capacitances of the anode and the cathode. The equivalent capacitance $C_{\text{diff}}$ represent reactants still present in the gas diffusion layer when the diffusion resistance increases. Here, the diffusion processes are combined for the anode and the cathode, since three time constants properly represent the dynamic behavior of the fuel cell stack.

$$U(i) = E_{\text{therm}} - i\left(R_{\text{ohmic}} + \frac{A_{\text{anode}} + B_{\text{anode}}\ln(i)}{i} + \frac{A_{\text{cathode}} + B_{\text{cathode}}\ln(i)}{i} + \frac{D\ln\left(\frac{i_{\text{limit}}}{i_{\text{limit}} - i}\right)}{i}\right), i = i_{\text{leak}} + i_{\text{load}} \tag{3}$$

Parameter estimation was performed using a curve fitting tool to estimate the parameters $E_{\text{therm}}$, $A_{\text{cathode}}$, $B_{\text{cathode}}$, $A_{\text{anode}}$, $B_{\text{anode}}$, $D$ and $i_{\text{leak}}$ from the measured polarization curves. The parameter $R_{\text{ohmic}}$ is obtained from the instantaneous voltage step response at high-frequency pulsed loading. The parameter $i_{\text{limit}}$ is obtained from measurement data as the maximum current in short-circuit conditions. The parameters $C_{\text{dl,anode}}$, $C_{\text{dl,cathode}}$ and $C_{\text{diff}}$ were estimated using a parameter estimation tool, comparing the simulated to the measured stack voltage response for identical input waveforms.

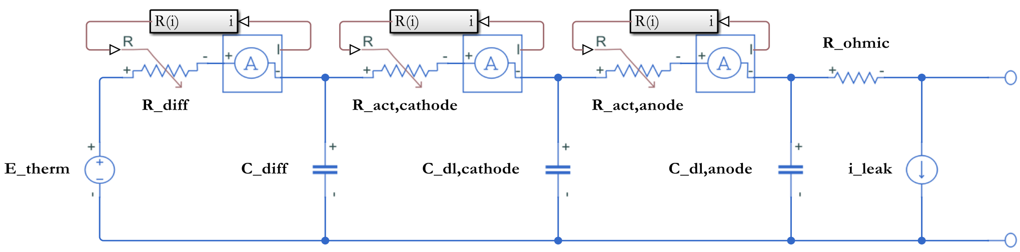


**Figure 3**. Dynamic PEM fuel cell equivalent circuit model in MATLAB/Simulink/Simscape.

Results of the simulations using the proposed model are shown in Figures 1 and 2. The fitted model shows a behavior closely following the measured data. The obtained and used set of parameters is listed in Table 1. The same model shows to be applicable for the entire frequency range from 1 Hz to 10 kHz. The proposed model can be easily fitted with measurement data obtained from basic measurements. Commonly used, but very fragile, electrochemical impedance spectroscopy measurements are not necessary, and constant phase elements in the equivalent circuit model are avoided.

**Table 1**. Estimated dynamic PEM fuel cell model parameter

| $E_{\text{therm}}$ | $R_{\text{ohmic}}$ | $A_{\text{cathode}}$ | $B_{\text{cathode}}$ | $C_{\text{dl,cath.}}$ | $A_{\text{anode}}$ | $B_{\text{anode}}$ | $C_{\text{dl,anode}}$ | $D$ | $C_{\text{diff}}$ | $i_{\text{leak}}$ | $i_{\text{limit}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 12.79 V | 20 mΩ | 2.45 V | 220 mV | 60 mF | 1.44 V | 110 mV | 30 mF | 1.1 V | 80 mF | 11 mA | 26 A |

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